Third Fractional Calculus Meeting

The aim of this meeting is to bring together senior and young researchers to interact and expose recent developments in Fractional Calculus, modelling, numerical analysis and efficient resolution of fractional differential equations and other related nonlocal problems.

This 3rd Meeting on Fractional Calculus is held at the University of Zaragoza during 25-27 September 2019 and it is organized and patronized by the Universidad de Zaragoza and the Institute of Mathematics and Applications.

The two previous meetings were held at the University Complutense of Madrid

- Workshop: 2nd Fractional Calculus Meeting, University Complutense of Madrid, 21-23 November, Madrid, 2018.
- Workshop: 1st Fractional Calculus Meeting, University Complutense of Madrid, 13-14 November, Madrid, 2017

The opening of the workshop and all the lectures of the workshop will take place in the Salon de Grados de la Facultad de Ciencias (Edificio A, Universidad de Zaragoza, Campus Plaza San Francisco, Pedro Cerbuna 12, Zaragoza). The programme includes plenary and contributed communications.

Reception of the conference participants and coffee breaks will be also in Facultad de Ciencias.

Zaragoza, September 2019

Luciano Abadías, José Luis Gracia and Pedro J. Miana

The Organizing Committee

IUMA and Universidad de Zaragoza, Spain

Scientific Committee

Jacky Cresson (Université de Pau et des Pays de l'Adour, France)

Carlos Lizama (Universidad de Santiago de Chile, Chile)

Luis Vázquez (IMI and Facultad de Informática, Universidad Complutense de Madrid, Spain)

Invited speakers

Eduardo Cuesta (Universidad de Valladolid, Spain)

Ernesto Estrada (Instituto de Matemáticas y Aplicaciones, Universidad de Zaragoza and ARAID Foundation, Spain)

José E. Galé (Instituto de Matemáticas y Aplicaciones, Universidad de Zaragoza, Spain)

Valentin Keyantuo (University of Puerto Rico)

Peter Massopust (Technische Universitat München, Germany)

Pablo Raúl Stinga (Iowa State University, USA)

Martin Stynes (Beijing Computational Science Research Center, China)

Luis Vázquez (IMI and Facultad de Informática, Universidad Complutense de Madrid, Spain)

THIRD FRACTIONAL CALCULUS MEETING, SEPTEMBER 2019, 25th-27th

TIME	WEDNESDAY 25	TIME	THURSDAY 26	TIME	FRIDAY 27
		09:00-10:00		09:00-10:00	ERNESTO ESTRADA
		10:00-11:00	PETER MASSOPUST	10:00-10:30	DIANA STAN
				10:30-11:00	JESUS OLIVA
		11:00-12:00	EDUARDO CUESTA	11:00-11:30	COFFEE BREAK
		12:30-13:30		11:30-12:30	PABLO R. STINGA
		12:30-13:00		12:30-13:00	CLOSING CEREMONY
15:30-15:45	REGISTRATION	13:00-15:30	LUNCH	13:00-15:30	LUNCH
15:45-16:00	OPENING CEREMONY	15:30-16:30	JOSÉ E. GALÉ	15:30-16:30	
16:00-17:00	LUIS VÁZQUEZ	16:30-17:00	JACKY CRESSON	16:30-17:30	
		17:00-17:30	TIM J. BURNS		
17:00-17:30	ÓSCAR CIAURRI	17:30-18:00	COFFEE BREAK	17:30-18:00	
17:30-18:00	COFFEE BREAK	18:00-19:00	VALENTIN KEYANTUO	18:00-19:00	
18:00-19:00	MARTIN STYNES			18:00-19:00	
19:00-19:30	ANN ALSAWOOR			18:00-19:00	

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

A Panoramic View of the Fractional Calculus. Applications

Luis VÁZQUEZ[1]

SUMMARY

The Fractional Calculus represent a natural instrument to model nonlocal phenomena either in space or in time. At the same time, the application of the Fractional Calculus shows an open scenario of deep mathematical issues as well as generalizations to relate different experimental contexts.

[1]Instituto de Matemática Interdisciplinar, Facultad de Informática, Universidad Complutense de Madrid, Spain
email: lvazquez@fdi.ucm.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Hardy inequalities for fractional operators

Óscar Ciaurri,

SUMMARY

The classical Hardy inequality

$$\int_0^\infty \left| \frac{1}{x} \int_0^x f(t)\,dt \right|^p \, dx \leq \left(\frac{p}{p-1} \right)^p \int_0^\infty |f(x)|^p \, dx, \qquad 1 < p < \infty,$$

is the first of a huge family of functional inequalities. There are extensions and generalizations of it in many different contexts. In this talk, we will focus on Hardy inequalities related to some fractional operators defined in terms of orthogonal polynomials. Moreover, we will show how they can be used to obtain some uncertainty principles and Pitt's inequalities.

[1]Department of Mathematics and Computation
University of La Rioja
email: oscar.ciaurri@unirioja.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

A widespread deficiency in current rigorous analyses of time-fractional initial-boundary value problems

Martin STYNES[1]

SUMMARY

Time-fractional initial-boundary value problems of the form $D_t^\alpha u - p\partial^2 u/\partial x^2 + cu = f$ are considered, where $D_t^\alpha u$ is a Caputo fractional derivative of order $\alpha \in (0,1)$. As $\alpha \to 1^-$, we prove that the solution u converges, uniformly on the space-time domain, to the solution of the classical parabolic initial-boundary value problem where $D_t^\alpha u$ is replaced by $\partial u/\partial t$. Nevertheless, most of the rigorous analyses of numerical methods for this time-fractional problem have error bounds that blow up as $\alpha \to 1^-$, as we demonstrate. We show that in some cases these analyses can be modified to obtain robust error bounds that do not blow up as $\alpha \to 1^-$.

[1]Beijing Computational Science Research Center, China
email: `m.stynes@csrc.ac.cn`

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Spectrum based exponential stability analysis of linear time-invariant fractional delay differential-algebraic systems

Ann ALSAWOOR, **Miloud SADKANE**[1]

SUMMARY

Spectrum-based exponential stability analysis is investigated for a fractional linear time-invariant delay differential-algebraic system of the form

$$\begin{aligned} ED_t^{\alpha}y(t) &= Ay(t)+By(t-\tau), \quad t\in[0,T], \quad 0<\alpha<1, \\ y(t) &= \varphi(t), \quad t\in[-\tau,0], \end{aligned}$$

where D_t^{α} denotes the Caputo fractional derivative of order α, E, A and B are square matrices of the same order with E singular, and τ is a delay parameter.

Sufficient conditions are presented to ensure the exponential stability for any delay parameter. It is shown that if the pair (E, A) is regular and of index 1, then the exponential stability holds independently of the delay. An example is provided to illustrate the effectiveness and applicability of the theoretical results

[1]Universit de Bretagne Occidentale, France
email: Ann.AlSawoorMaskoni@univ-brest.fr, Miloud.Sadkane@univ-brest.fr

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Splines Meet Fractional Operators

Peter Massopust[1]

SUMMARY

We present relations between generalizations of classical cardinal B-splines and certain fractional difference and differential operators. These relations allow the inclusion of generalized splines into the area of fractional calculus.

References

[1] P. Massopust. Fractional operators, Dirichlet averages, and splines. In *Applied and Numerical Harmonic Analysis: New Perspectives on Approximation and Sampling Theory (Festschrift in Honor of Paul Butzer's 85th Birthday)*, Ahmed I. Zayed and Gerhard Schmeisser (eds.) , pp. 399–422. Birkhäuser, Basel, 2014.

[1]Center of Mathematics
Technical University of Munich
email: massopust@ma.tum.de

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Nonlinear Fractional Equations: Optimal Hölder Regularity. Applications to A Posteriori Error Estimation

Eduardo CUESTA[1]

SUMMARY

The regularity of the solutions of differential equations (in L^p sense, in the sense of the existence of certain number of derivatives, optimal regularity,...) attracted the interest of researchers for decades.

In this talk we will present recent results related to the optimal regularity of nonlinear fractional evolution equation. In fact our contribution is twofold: On the one hand we study the optimal regularity, in the sense of Hölder and weighted–Hölder continuity, for the solutions of abstract nonlinear evolution equations involving fractional derivatives (or integrals) in time, i.e.

$$(1) \qquad \partial_t^\alpha u(t) = F(t, u(t)), \qquad t > 0,$$

where $1 < \alpha < 2$, and $F : X \to X$ is a nonlinear functional defined in a complex Banach space X; On the other hand we state a theoretical framework where such a regularity results allow to provided a posteriori error estimates for a large variety of time discretization of (1).

We must highlight that since the results shown in this talk have been obtained in the general/asbtract framework of complex Banach spaces, they can be applied for a wide range of equations (and its time discretzations) and applications.

[1]Universidad de Valladolid, Spain
email: eduardo@mat.uva.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

On domains of functions of Cesàro operators. Poisson equation and discrete Hilbert transform.

José E. Galé[1]

SUMMARY

We study domains of (unbounded) operatorial functions related with bounded Cesàro operators of fractional order, with application to the Poisson equation and the discrete, one-sided, Hilbert transform. Our main tools are a functional calculus introduced in terms of finite difference operators and the sequence of Cesàro numbers, and a notion of admissibility for holomorphic functions which allows us to widen that functional calculus suitably. In the way, we establish some new results on ergodicity and give a generalization of Kaluza's theorem on log-convex sequences. In particular, the results obtained in this paper provide an extension of methods introduced by Haase and Tomilov in the subject. Examples involving the Volterra integral operator and the backward shift operator are included to illustrate the theory.

[1]Departamento de Matemáticas, Instituto Universitario de Matemáticas y Aplicaciones, Universidad de Zaragoza, Spain.
email: gale@unizar.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

The Noether's theorem in the fractional calculus of variations

Jacky CRESSON[1]

SUMMARY

Recently, many authors have tried to extend the classical Noether's theorem in the context of the fractional calculus of variations. In this talk, we will discuss the difficulties associated to such a generalization and we will formulate and prove a general Noether's type theorem for fractional Lagrangian systems. Numerical simulation supporting our result will be given.

[1]Universit de Pau et des Pays de l'Adour, France
email: jacky.cresson@univ-pau.fr

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Estimation of the Derivative and Fractional Derivative of a Function Specified by Noisy Data

Timothy J. Burns, **Bert W. Rust**[1]

SUMMARY

We present a method for estimating the derivative and the fractional derivative of a smooth function that is defined by a finite set of data measurements, which have been contaminated by noise, with known statistical properties. Key assumptions are that the measurements are from a process which preferentially damps high-frequency content in the data, and the residual corresponding to the noise is a realization of a white noise time series. Our method uses some ideas originally developed by Rust [1] for approximating the solution of first-kind Fredholm integral equations given noisy data, and a new approach by Zhao, et al. [2] to solving the first-kind Volterra integral equation that corresponds to differentiation. The method takes advantage of known singular value decompositions of the associated compact Volterra integral operators of differentiation and fractional differentiation on and into weighted L_2 spaces [2],[3].

References

[1] B. W. Rust. Truncating the singular value decomposition for ill-posed problems. Technical Report NISTIR 6131, National Istitute of Standards and Technology, Gaithersburg, MD, July 1998.

[2] Z. Zhao, Z. Meng, and G. He. A New approach to numerical differentiation. *J. Comput. Appl. Math.* **232**, 227–239, 2009.

[3] R. Gorenflo and K. V. Tuan. Singular value decomposition of fractional integration operators in L_2-spaces with weights. *Journal of Inverse and Ill-posed Problems*, **3**, 1–10, 1995.

[1]Applied & Computational Mathematics Division
National Institute of Standards and Technology
Gaithersburg, MD, USA

email: `timothy.burns@nist.gov`

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Analytic semigroups and fractional calculus

Valentin Keyantuo

SUMMARY

We consider some analytic semigroups which appear in the study of fractional differential equations. The first is related to the Hadamard fractional integral and the averaging operator $Cf(x) = \frac{1}{x}\int_0^x f(t)dt$ acting on the space $L^p[0, 1]$, $1 \leq p < \infty$. The second semigroup we study is the semigroup generated by the fractional Laplace operator with exterior Dirichlet condition. Analyticity in of the semigroup $L^p(\Omega)$ where Ω is an open subset of $\mathbb{R}^N$ is proved.

The talk is based on joint work with Ahmed Sani, Omar Elmennaoui, Fabian Seoanes and Mahamadi Warma.

[1]Department of Mathematics
University of Puerto Rico, Río Piedras Campus
email: `valentin.keyantuo1@upr.edu`

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

d-path Laplacian operators in graphs and networks

Ernesto ESTRADA[1]

SUMMARY

The discrete Laplacian operator is related to the most important dynamical processes on graphs and networks. Here I generalize this operator to account for the hopping of a diffusive particle to non-nearest neighbours in a graph. I will present analytical results about the boundness and self-adjoitness of these operators as well as their Laplace and Mellin transform. Using such transformed operators I will introduce a generalized diffusion equation on graphs. I will prove that the Mellin transformed d-path Laplacian operators induces superdiffusive processes on graphs for certain range of Mellin parameters. However, the diffusion controlled by the Laplace transformed d-path Laplacians is never anomalous. Finally, I will show some areas of applications of these results to the study of real-world networks.

[1]Instituto Universitario de Investigación de Matemáticas y Aplicaciones (IUMA),
Universidad de Zaragoza, Spain
email: estrada66@posta.unizar.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

Porous medium equations with nonlocal pressure

Diana STAN[1]

SUMMARY

We provide a description of the results obtained so far on the nonlinear diffusion equation $u_t = \nabla \cdot (u^{m-1}\nabla(-\Delta)^{-s}u)$, which describes a flow through a porous medium driven by a nonlocal pressure. We consider constant parameters $m > 1$ and $0 < s < 1$, we assume that the solutions are non-negative, and the problem is posed in the whole space. We present a theory of existence of solutions, results on uniqueness, asymptotic behavior and relation to other models.

This work is in collaboration with Félix del Teso (BCAM, Bilbao) and Juan Luis Vázquez (UAM, Madrid).

References

[1] Stan, Diana; del Teso, Félix; Vázquez, Juan Luis. Chapter: Porous medium equation with nonlocal pressure. Springer Book: "Current Research in Nonlinear Analysis: In Honor of Haim Brezis and Louis Nirenberg", 2018, 277–308.

[2] Stan, Diana; del Teso, Félix; Vázquez, Juan Luis. Existence of weak solutions for a general porous medium equation with nonlocal pressure. *Archive for Rational Mechanics and Analysis,* (2019) vol. 233, Issue 1, pp 451–496.

[1]Departamento de Matemáticas, Estadística y Computación, Universidad de Cantabria, Spain
email: diana.stan@unican.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

A family of integral operators on spaces of fractional order

Miana, Pedro J.[1], **Oliva-Maza, Jesús**[2]

SUMMARY

The Sobolev-Lebesgue spaces $\mathcal{T}_p^\alpha(t^\alpha)$ ($\alpha \geq 0$, $p \geq 1$), which are embedded on $L^p(\mathbb{R}^+)$, were first introduced in [1]. They consist of functions with derivatives of fractional order α, obtained by Weyl fractional calculus.

In this talk, we present our recent work concerning these $\mathcal{T}_p^\alpha(t^\alpha)$ spaces. More specifically, we study how the three-parameter family of integral operators $\mathcal{S}_{\beta,\mu,\lambda}$, given by:

$$\mathcal{S}_{\beta,\mu,\lambda} f(t) = t^{\mu\lambda-\beta} \int_0^\infty \frac{s^{\beta-1}}{(s^\lambda + t^\lambda)^\mu} ds, \qquad t \geq 0,$$

acts on $\mathcal{T}_p^\alpha(t^\alpha)$, that is, when $f \in \mathcal{T}_p^\alpha(t^\alpha)$. This family of operators generalizes both the so-called Stieltjes transform $\mathcal{S} = \mathcal{S}_{1,1,1}$ and the Poisson transform $\mathcal{P} = \mathcal{S}_{2,2,1}$.

In addition, we also give some new results regarding the Hölder's inequality and the Hilbert transform on $\mathcal{T}_p^\alpha(t^\alpha)$.

References

[1] J. J. Royo. Convolution algebras and modules on $\mathbb{R}^+$ defined by fractional derivative, *PhD. Thesis, Universidad de Zaragoza*, 2008.

[1]Departamento de Matemáticas
Universidad de Zaragoza
pjmiana@unizar.es

[2]Departamento de Matemáticas
Universidad de Zaragoza
joliva@unizar.es

3rd Fractional Calculus Meeting
Zaragoza, September 25–27 2019

The one-sided nature of fractional derivatives

Pablo Raúl Stinga[1]

SUMMARY

It is well-known that fractional derivatives are one-sided in nature. Indeed, Marchaud, Caputo, Riemann–Liouville and other fractional derivatives look at the values of functions either to the left or to the right of a point. On the other hand, classical Sobolev spaces are *two-sided*, and this is because test functions are differentiable. We will report on a recently developed theory of one-sided Sobolev spaces that is well-suited for capturing the underlying one-sided structure of classical derivatives, fractional derivatives, and the Fundamental Theorem of (Fractional) Calculus. These are joint works with A. Bernardis, F. J. Martín-Reyes and J. L. Torrea [1] and my PhD student Mary Vaughan [2].

References

[1] A. Bernardis, F. J. Martín-Reyes, P. R. Stinga and J. L. Torrea. Maximum principles, extension problem and inversion for nonlocal one-sided equations. *J. Differential Equations* **260**, 6333–6362, 2016.

[2] P. R. Stinga and M. Vaughan. One-sided fractional derivatives, fractional Laplacians and weighted Sobolev spaces. *Nonlinear Anal.* to appear, 2019.

[1]Department of Mathematics, Iowa State University, USA
email: stinga@iastate.edu

List of participants

Ann Alsawoor (Université de Bretagne Occidentale, France)

Luciano Abadías (IUMA & Universidad de Zaragoza, Spain)

Óscar Carrión (Universidad de Zaragoza, Spain)

Óscar Ciaurri (Universidad de la Rioja, Spain)

Carmelo Clavero (IUMA & Universidad de Zaragoza, Spain)

J. Alberto Conejero (Universidad Politécnica de Valencia, Spain)

Jacky Cresson (Université de Pau et des Pays de l'Adour, France)

Eduardo Cuesta (Universidad de Valladolid, Spain)

Isabel Echeverribar(Universidad de Zaragoza, Spain)

Ernesto Estrada (IUMA, Universidad de Zaragoza & ARAID Foundation, Spain)

José E. Galé (IUMA & Universidad de Zaragoza, Spain)

Geovanny Gordillo (Universidad de Zaragoza, Spain)

José Luis Gracia (IUMA & Universidad de Zaragoza, Spain)

Salvador Jiménez (IMI & Universidad Complutense de Madrid, Spain)

Valentin Keyantuo (University of Puerto Rico)

Álvaro Lozano (IUMA & Centro Universitario de la Defensa, Zaragoza, Spain)

Sergio Martínez (Universidad de Zaragoza, Spain)

Peter Massopust (Technische Universitat München, Germany)

Pedro J. Miana (IUMA & Universidad de Zaragoza, Spain)

Adrián Navas (Universidad de Zaragoza, Spain)

Jesús Oliva-Maza (IUMA & Universidad de Zaragoza, Spain)

Luis Rández (IUMA & Universidad de Zaragoza, Spain)

Pablo Solan (Universidad de Zaragoza, Spain)

Pablo Raúl Stinga (Iowa State University, USA)

Diana Stan (Universidad de Cantabria, Spain)

Martin Stynes (Beijing Computational Science Research Center, China)

Timothy J. Burns (National Institute of Standards and Technology Gaithersburg, MD, USA)

Luis Vázquez (IMI & Facultad de Informática, Universidad Complutense de Madrid, Spain)

Raquel Villacampa (IUMA & Centro Univversitario de la Defensa, Zaragoza, Spain)

Instituto Universitario de Investigación de Matemáticas y Aplicaciones Universidad Zaragoza

Facultad de Ciencias Universidad Zaragoza

Universidad Zaragoza 1542

GOBIERNO DE ARAGON