

# IUMA day on Special Structures in Geometry

(Zaragoza, 26 June 2014)

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Place: Aula 9, Edificio de Matemáticas, Universidad de Zaragoza

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## Deformations of $G_2$ -structures

*Sergey Grigorian*

(University of Texas-Pan American, Edinburg, USA)

ABSTRACT. We consider non-infinitesimal deformations of  $G_2$ -structures on 7-dimensional manifolds and derive an exact expression for the torsion of the deformed  $G_2$ -structure. We then specialize to a case where the deformation is defined by a vector  $v$  and we explicitly derive the expressions for the different torsion components of the new  $G_2$ -structure in terms of the old torsion components and derivatives of  $v$ . In particular this gives a set of differential equations for the vector  $v$  which have to be satisfied for a transition between  $G_2$ -structures with particular torsion classes. For some specific torsion classes we find that these equations have no solutions.

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## Elementary deformations and the HK/QK correspondence

*Oscar Macià*

(Universidad de Valencia)

ABSTRACT. The HK/QK correspondence constructs quaternionic Kaehler metrics from hyperkaehler metrics with a rotating circle symmetry. We will discuss how this may be interpreted as a combination of Swann's twist construction with the concept of elementary deformation, leading to a uniqueness statement for the HK/QK correspondence, and will see how basic examples of the c-map may be realised in this way.

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## Formality in Co-Symplectic and Sasakian Geometries

*Marisa Fernández*

(Universidad del País Vasco)

ABSTRACT. In this talk, we study the formality of the mapping torus of an orientation-preserving diffeomorphism of an oriented compact differentiable manifold. In particular, we give conditions under which a mapping torus, not necessarily symplectic, has a non-zero Massey product. We apply this to prove that there are compact non-formal co-symplectic manifolds of dimension  $m (= 2n + 1)$  and with first Betti number  $b$  if and only if  $m = 3$  and  $b \geq 2$ , or  $m \geq 5$  and  $b \geq 1$ . On the other hand, we prove that all higher Massey products on any simply connected compact Sasakian manifold vanish. However, for every  $n \geq 3$ , we exhibit the first examples of simply connected compact regular Sasakian manifolds of dimension  $2n + 1$  which are non-formal because they have a non-zero triple Massey product.

Joint work with G. Bazzoni, I. Biswas, V. Muñoz and A. Tralle.

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## Left-invariant conformal Killing-Yano 2-forms on Lie groups

Adrián Andrada

(FaMAF-CIEM, Universidad Nacional de Córdoba, Argentina)

ABSTRACT. A  $p$ -form  $\omega$  on an  $n$ -dimensional Riemannian manifold  $(M, g)$  is called *conformal Killing-Yano* if it satisfies

$$\nabla_X \omega = \frac{1}{p+1} \iota_X d\omega - \frac{1}{n-p+1} X^* \wedge d^* \omega, \quad X \in \mathfrak{X}(M),$$

where  $\nabla$  is the Levi-Civita connection,  $X^*$  is the 1-form dual to  $X$  and  $d^*$  is the co-differential. If, moreover,  $\omega$  is co-closed, that is  $d^* \omega = 0$ , then it is called *Killing-Yano*.

These objects were introduced in [3, 5] and are natural generalizations of Killing vector fields and conformal Killing vector fields; they have had many applications in mathematical physics (see for instance [2, 4]). A conformal Killing-Yano form with  $d^* \omega \neq 0$  is called *strict*. For instance, any Sasakian manifold admits CKY 2-forms, given by  $d\xi^*$ , where  $\xi$  is the corresponding Reeb vector field.

In this work we study Lie groups equipped with left-invariant Riemannian metrics that admit left-invariant conformal Killing-Yano 2-forms. Firstly we show that if a Lie group admits a strict CKY 2-form, then the dimension of the group is odd. We also provide a construction to obtain examples of left-invariant CKY 2-forms on Lie groups beginning with an even-dimensional Lie group equipped with a non-degenerate parallel 2-form.

Next we provide the classification of all 3-dimensional Lie groups equipped with such tensors, and we obtain that in this dimension any left-invariant KY tensor is parallel. We show that if a nilpotent Lie group admits a strict conformal Killing-Yano 2-form, then the group is isomorphic to the Heisenberg group  $H_{2n+1}$ . In the compact case, we show that the only compact Lie group equipped with a bi-invariant metric that admits strict CKY 2-forms is  $SU(2)$ . Moreover, a left-invariant metric  $g$  on  $SU(2)$  admits such 2-forms if and only if  $g$  is homothetic to a Berger metric on the sphere  $S^3$ . This work extends results in [1], where Killing-Yano 2-forms on Lie groups were studied.

This is a joint work with Isabel Dotti and María Laura Barberis (U.N. Córdoba).

### REFERENCES

- [1] M.L. Barberis, I. Dotti, O. Santillán, The Killing-Yano equation on Lie groups, *Class. Quantum Grav.* **29** (2012) 065004 (10pp).
- [2] I.M. Benn, P. Charlton, Dirac symmetry operators from conformal Killing-Yano tensors, *Class. Quantum Grav.* **14** (1997), 1037–1042.
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- [4] R. Penrose, M. Walker, On quadratic first integrals of the geodesic equations for type {22} spacetimes, *Comm. Math. Phys.* **18** (1970), 265–274.
- [5] K. Yano, Some remarks on tensor fields and curvature, *Ann. of Math.* **55** (1952), 328–347.

## Six dimensional solvmanifolds with holomorphically trivial canonical bundle

Antonio Ota

(Centro Universitario de la Defensa Zaragoza, IUMA)

ABSTRACT. We determine the 6-dimensional solvmanifolds admitting an invariant complex structure with holomorphically trivial canonical bundle. Such complex structures are classified up to isomorphism, and the existence of strong Kähler with torsion (SKT), generalized Gauduchon, balanced and strongly Gauduchon metrics is studied. We show several holomorphic deformations with interesting behaviours.

## Embedded quaternionic contact hypersurfaces

Dimitar Vassilev

(University of New Mexico, Albuquerque, New Mexico, USA)

ABSTRACT. We present results on quaternionic contact hypersurfaces embedded in hyper-Kähler spaces, including a full description in the case of the flat quaternion ambient space.