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# Towards Poroelasticity: Variational Space-Time Methods for Elastic Waves and Single-Phase Flow

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**ABSTRACT**

Poroelasticity has become of increasing importance in a diverse range of engineering fields. Vivid examples remind that the effect of fluid-induced deformations and fluid-solid interactions cannot be ignored in these applications. Therefore, the ability to simulate coupled mechanical deformations and flow in porous media phenomena is of particular importance from the point of view of physical realism. However, the numerical simulation of coupled processes of mechanical deformations and fluid flow remains a challenging task due to the complex structure of the model equations.

Here we present and study three families of continuous, discontinuous and $C^1$-continuous Galerkin time discretization schemes for the simulation of elastic wave propagation and single-phase porous media flow. Variational time discretization schemes are getting of increasing importance for the numerical approximation of transient processes since they offer appreciable advantages like the natural construction of higher order methods and the applicability of adaptive finite element techniques. Together with discontinuous Galerkin discretizations of the spatial variables they are used for simulating elastic wave propagation in multiscale material (cf. [1]),

$$\rho\,\partial_t^2 \boldsymbol{u} - \nabla\cdot\boldsymbol{\sigma}(\boldsymbol{u}) = \boldsymbol{f}\,, \qquad \boldsymbol{\sigma} = \boldsymbol{C}(\boldsymbol{x}) : \boldsymbol{\epsilon}\,, \quad \boldsymbol{\epsilon} = \left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^\top\right)/2\,. \tag{1}$$

Further, we study parabolic transport in porous media written in the mixed form

$$\partial_t u + \nabla\cdot\boldsymbol{q} = f\,, \qquad \boldsymbol{q} = -\boldsymbol{D}\nabla u\,. \tag{2}$$

Here, the Galerkin time discretizations are combined with mixed finite element methods in space; cf. [1]. For (2), an analysis of the fully discrete scheme is presented. For (1) and (2) an unified framework for the efficient solution of the resulting algebraic block systems of equations is addressed. The performance properties of the schemes are illustrated by numerical experiments in anisotropic and heterogenous media.

Finally, we demonstrate the applicability of the presented methods as building blocks for the simulation of poroelasticity described by Biot's consolidation model.

## References

[1] U. Köcher, *Variational space-time methods for the elastic wave equation and the diffusion equation*, PhD Thesis, Helmut Schmidt University, Hamburg, http://edoc.sub.uni-hamburg.de/hsu/volltexte/2015/3112/, (2015).

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